

Calculator are not allowed

Answer the following questions:

Q. 1 Let $y = \frac{100}{\sin x + 5 \cos x}$ If x changes from 45° to 46° , show that .

$$\Delta y \approx \frac{5\sqrt{2}}{81}\pi$$

Q. 2 Find the equation of the tangent line to the graph of

$$(x^2 + y^2)^2 = 4xy, \quad \text{at the point } P(1, 1). \quad (7 \text{ pts})$$

Q. 3 Show that the equation

$$x^3 + 3x + 1 = 0$$

has exactly one real root.

(7 pts)

Q. 4 Show that the volume of the right circular cylinder of largest volume that can be inscribed in a given right circular cone is $\frac{4}{9}$ the volume of the cone.

(7 pts)

Q. 5 Let $f(x) = 4x + \frac{1}{x}$.

- Find the vertical and horizontal asymptotes for the graph of f (if any).
- Find the intervals on which f is increasing or decreasing, and find the local extrema of f (if any).
- Find the intervals on which the graph of f is concave upward or concave downward, and find the points of inflection (if any).
- Sketch the graph of f .

(12 pts)

(Good Luck)